Pattern formation in nature:
Physical constraints and self-organising characteristics

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Abstract

The formation of patterns is apparent in natural systems ranging from clouds to animal markings, and from sand dunes to the intricate shells of microscopic marine organisms. Despite the astonishing range and variety of such structures, many seem to have analogous features: the zebra's stripes put us in mind of the ripples of blown sand, for example. In this article I review some of the common patterns found in nature and explain how they are typically formed through simple, local interactions between many components of a system – a form of physical computation that gives rise to self-organisation and emergent structures and behaviours.

Introduction

When the naturalist Joseph Banks first encountered Fingal's Cave on the Scottish island of Staffa, he was astonished by the quasi-geometric, prismatic pillars of rock that flank the entrance. As Banks put it,

> Compared to this what are the cathedrals or palaces built by men! Mere models or playthings, as diminutive as his works will always be when compared with those of nature. What now is the boast of the architect! *Regularity*, the only part in which he fancied himself to exceed his mistress, Nature, is here found in her possession, and here it has been for ages undescribed.

This structure has a counterpart on the coast of Ireland: the Giant's Causeway in County Antrim, where again one can see the extraordinarily regular and geometric honeycomb structure of the fractured igneous rock (Figure 1).

When we make an architectural pattern like this, it is through careful planning and construction, with each individual element cut to shape and laid in place. Our experience from human technologies thus suggests that making a pattern requires a patterner. But at Fingal's Cave and the Giant's Causeway, the forces of nature have conspired to produce a pattern without, we must presume, any blueprint or foresight or design. This is an example of spontaneous pattern formation.[1,2]

In earlier times, such regularity in nature was taken as evidence of God's guiding hand. We now know that no intelligent agency is needed to create the patterns that appear profusely in both the living and the inorganic natural world. These organised arrays of elements arise spontaneously from the interactions between their many component parts, whether these are for example chemical reagents that react and diffuse, small particles or molecules that cohere into clusters, propagating cracks, wind-blown sand grains or flowing liquids. Such patterns are said to be *self-organised*. Their scientific study comprises one aspect of research into so-called *complex systems*, which typically show *emergent* behaviours that cannot be deduced or predicted by a focus on the properties of the individual elements. Such regularities are not just a feature of the insensate or instinctive natural world, but may also be found in human social systems that are seemingly subject to the whims of free will – for example, in the evenly spaced waves of congestion that might appear in moving traffic, or quasi-periodic cycles in economic systems. Understanding how spontaneous pattern formation occurs is therefore an endeavour that unites many disparate fields of science, from zoology to fracture mechanics, and from chemical kinetics to sociology. Many patterns in nature have a universal aspect that does not respect the traditional divisions between the natural sciences, or even between the living and the non-living world. Rather, natural patterns seem to come from a relatively limited palette, even in systems that might seem to have nothing at all in common with one another. The hexagonal columns of Fingal's Cave may put us in mind of other natural hexagonal forms (Figure 2). Do these patterns really have anything in common, or is the similarity in appearance just coincidence?

The first person to confront this question in a systematic way was the Scottish zoologist D'Arcy Wentworth Thompson in his book *On Growth and Form* (1917)[3], which collected together all that was then known about pattern and form in nature in a synthesis of biology, natural history, mathematics, physics and engineering. Thompson pointed out that, in biology at least, and often in the nonliving world, pattern formation is not a static thing but arises from growth: 'Everything is what it is because it got that way.' The answer to the riddle of pattern lies in how it got to be that way. That is less obvious than it sounds: a bridge or a paddy field or a microchip is 'explained' by how it looks, not how it was made. Thompson objected to the fact that Darwinian, adaptive explanations of form and pattern in living systems tended to take that approach too. The zebra's stripes might be 'explained' in one sense by invoking the adaptive benefit of their camouflage (although in fact the effectiveness of the stripes for concealment is still debated) – but this does not account for how any particular zebra acquires these pigmented markings on its hide as it grows from an embryo. Thompson argued that evolutionary biology needs to take into account both the limitations imposed and the possibilities provided by purely physical forces (including chemical processes) acting on a growing organism.

It is now understood that the common features of many natural patterns result from mathematical analogies and equivalences in the rules governing their formation – whether these rules are expressed in terms of continuum equations describing, say, diffusion and transport of material, or as local interactions between the components. Both descriptions tend to give rise to solutions that involve symmetry-breaking of an initially uniform or random system, creating spatial and/or temporal segregation of the constituents. These solutions are often modified by specific circumstances such as boundary conditions, the appearance of defects, and path-dependent hysteresis in transitions between patterns. At root, however, they expose self-organisation as an inherent property of complex systems.

Chemical patterns

An explanation for the patterning apparent in animal markings – noted but barely explored by D'Arcy Thompson – has emerged from the study of so-called oscillating chemical reactions, in which a mixture of chemical reagents seems first to react in one direction and then to reverse and reform the original mixture.[4,5] The classic example is the Belousov-Zhabotinsky (BZ) reaction discovered in the 1960s, in which the mixture alternates between red and blue owing to changes in the charge state of the iron ions that catalyse the reactions. If these oscillations went on indefinitely, the system would violate the second law of thermodynamics. But eventually they die out and the mixture settles into an equilibrium state. Thermodynamics is silent about the progress of a chemical reaction, but pronounces only on its final, stable state. The oscillations occur only so long as the system is *out of equilibrium*. It can be maintained in that state indefinitely, however, by continually feeding in fresh reagents and carrying away the end products.

If the mixture is stirred, the colour change takes place more or less everywhere at once. But if the reaction proceeds in an undisturbed thin layer of liquid – or better still, a gel, to slow diffusion and suppress fluid-dynamic disturbances – parts of it can undergo the switch at different times. This does not just produce a patchwork, but regular patterns of concentric bands and spirals: chemical waves that spread through the medium, like the expanding ripples of a stone thrown into a pond (Figure 4). Where two wavefronts meet, they annihilate each other.

The oscillations arise because the reaction has two possible states or branches. Each branch involves an autocatalytic process: the reaction products promote the formation of more of themselves. This is a positive feedback process, and it means that each branch eventually exhausts itself as the feedback speeds up the reaction and exhausts the ingredients. When that happens, the system becomes primed to switch to the other branch. The oscillating process depends on two general factors: the ingredients are reacting to produce the coloured products, and the molecules involved are moving by diffusion through the liquid. How quickly the system switches between the red and blue states then depends on how quickly diffusion brings in fresh ingredients to replenish those that have been lost by reaction. So the patterns here come from a balance between reaction, which destroys the ingredients, and diffusion, which replenishes them. This kind of process is known as a reaction-diffusion system. Notice that the pattern comes about through a competition between these two opposing processes. That is one of the universal principles of pattern formation: it requires a balance of opposite tendencies.

In 1952 Alan Turing identified another kind of chemical reaction-diffusion process, which produces not moving chemical waves but *stationary* patterns.[6] Turing's theory was prompted by the question of how pattern formation occurs in the earliest stages of embryo formation on a fertilised egg. This is a problem of symmetry breaking: a spherically symmetrical ball of identical cells becomes differentiated into those that will form the head, the limbs, the spine and so forth. Turing suggested that the patterns could be created by two types of chemical substance (morphogens or 'shape-formers'), both of them diffusing through the system. One is called an activator, because it catalyses its own formation: it is again autocatalytic. The other morphogen is a substance that interferes with this self-generation by the activator: it is, in other words, an inhibitor.[7] Turing showed that stationary patterns – persistent differences in the concentrations of the activator and inhibitor from one part of the system to another – can arise if the inhibitor diffuses more quickly than the activator.

Turing's model generates two particular kinds of pattern: spots and stripes (Figure 4*a*). These predictions were verified when Turing patterns were first produced experimentally in the 1990s.[8,9] The theory suggests a mechanism for animal markings: during embryo growth, diffusing morphogens imprinting the skin with patterns that either switched on pigment-generating genes or left them off.[10,11] Turing-type models can explain many of the features of animal markings, for example on fish[12], wildcats[13] and ladybirds[14] (Figure 4*b*). Specific morphogens responsible for these patterns have not yet been identified, but they have in the analogous case of the regular positioning of hair follicles in mammals.[15]

Granular patterns

The stripes of the zebra might put us in mind of the patterns in windblown sand (Figure 5*a*).[16] It has been argued that the formation of sand ripples and dunes can also be regarded as a process localised activation and longer-ranged suppression. The appearance of a ripple is a self-activating or autocatalytic process: as soon as a tiny bump grows on a flat surface that is being scattered with wind-blown sand, it starts to capture more grains than the surrounding surface, and so it grows bigger.[17] And the bigger it gets, the more grains it captures. At the same time, this means that the wind gets depleted of its grains, and so there is less chance of another ripple developing in the lee of an existing one: there is inhibition around the ripple, so that the next ripple has to be a certain minimum distance away.

Dunes are created in a similar process, modified by factors such as wind dynamics, local topography and vegetation. Sand may become self-organised in this way into a variety of structures, including straight and undulating (seif) ripple dunes, crescent-shaped barchan dunes and many-armed star dunes. On the granular surface of Mars, differences in gravity, atmospheric pressure and wind speed give rise to some quite new types of dune shape not seen on Earth (Figure 5*b*).

Systems of interacting grains can form a wide variety of patterns. For example, a very thin layer of spherical metal grains vertically vibrated in a shallow, sealed and evacuated container will form stationary waves called oscillons in which the grains are constantly rising and falling in step with each other (Figure 6*a*).[18,19] These waves will become self-organised into regular arrays of stripes, spirals, hexagonal and square cells, and more random, non-stationary cell-like patterns that appear to be turbulent (Figure 6*b*). The pattern selected depends on the frequency and amplitude of the shaking, and switches between patterns happen abruptly as critical thresholds are crossed. These patterns result from collisions between grains, which put the grains literally ‘in touch’ with one another so that their movements may become synchronised. The patterns can be reproduced in a model which assumes merely that the grains lose a little energy when they collide.[20]

If such ‘grains’ may move of their own accord, and interact via relatively simple rules of attraction, repulsion and mimicry, as some animal populations do, they can display the kinds of coordinated swarming patterns seen for fish and birds.[21,22] Simple models based on the emergent behaviour of many interacting particles have also been used to account for branching and aggregation patterns, which may be as regular as those of snowflakes[23] or as apparently disorderly as those of river networks[24] and cities[25].

Patterns as computation

There is – despite aspirations to the contrary – no universal theory of pattern formation in nature. Nonetheless, it has proved possible to identify many common principles, such as the universality of certain basic forms (hexagons, stripes, hierarchical branches, fractal shapes, spirals…), the importance of non-equilibrium growth processes and of a balance or to-and-fro between conflicting driving forces, and the existence of sharp thresholds of driving force that produce global changes in the pattern. The pseudo-hexagonal cracks of Fingal's cave and the Giant's Causeway seem to emerge as a near-optimal way of releasing the tension that builds up in molten rock as it cools, hardens and contracts: an orderly energetic minimum selected from an initially random arrangement of cracks in an iterative manner as the cracks descend through the solidifying material,[26] balancing forces in a manner – and with a result – not unlike that found in foams.[27]

In general, self-organised patterns can be regarded as a kind of computation performed by the interactions of physical particles. This is made most apparent in models based on cellular automata: discrete elements (cells) organised on a regular grid, which interact via simple, local rules that depend on the state of neighbouring cells. Cellular automata were first invoked by John von Neumann and Stanislas Ulam in the 1950s within the context of a generalised theory of computation: each cell can be considered as a 'memory' cell encoding information in its physical state. Von Neumann was interested in whether such automata might be able to replicate patterns of information and thereby to evolve into more complex computational states, ultimately displaying a form of 'thinking'. The capacity of cellular automata to spawn complex patterns that can replicate or move across the grid was revealed in the Game of Life, a particular cellular automaton devised in the 1960s by the mathematician John Horton Conway. The connections both with computation and with self-organised patterns has been extensively investigated by Wolfram.[28] The patterns I have described here, including chemical waves,[29] can be reproduced in models based on cellular automata rather than, for example, continuum equations of chemical diffusion and kinetics. This illustrates how spontaneous patterning is a general property of complex systems of many interacting components, interacting via local rules that are often relatively simple.

In the living world pattern formation seems both to constrain adaptive change and to offer new adaptive opportunities – to operate, in other words, in parallel and sometimes in sympathy with Darwinian evolution. The technological and aesthetic possibilities of spontaneous pattern formation, for example in materials science, architecture and the production of structurally and dynamically complex chemical systems, is only just beginning to be explored.

References

1. Philip Ball, *The Self-Made Tapestry*, Oxford University Press (Oxford), 1999.

2. Philip Ball, *Nature's Patterns*, Oxford University Press (Oxford), 2009.

3. D'Arcy Wentworth Thompson, *On Growth and Form*, Cambridge University Press (Cambridge), 1917.

4. Arthur Winfree, *When Time Breaks Down*, Princeton University Press (Princeton), 1987.

5. Stephen K Scott, *Oscillations, Waves and Chaos in Chemical Kinetics*, Oxford University Press (Oxford), 1994.

6. AM Turing, *Phil. Trans. R. Soc. B* **237**, 37, 1952.

7. A Gierer & H Meinhardt, *Kybernetik* **12**, 30, 1972.

8. V Castets, E Dulos, J Boissonade & P De Kepper, *Phys. Rev. Lett.* **64**, 2953, 1990.

9. Q Ouyang & HL Swinney, *Nature* **352**, 610, 1991.

10. James D Murray, *Mathematical Biology*, Springer (Berlin), 1990.

11. Hans Meinhardt, *Models of Biological Pattern Formation*, Academic Press (London), 1982.

12. S Kondo & R Asai, *Nature* **376**, 765 (1995).

13. RT Liu, SS Liaw & PK Maini, *Phys. Rev. E* **74**, 011914, 2006.

14. SS Liaw, CC Yang, RT Liu & JT Hong, *Phys. Rev. E* **64**, 041909 (2001).

15. S Sick, S Reinker, J Timmer & T Schlake, *Science* **314**, 1447, 2006.

16. Ralph A Bagnold, *The Physics of Wind-Blown Sand and Desert Dunes*, Methuen (London), 1941.

17. SB Forrest & PK Haff, *Science* **255**, 1240, 1992.

18. PM Umbanhowar, F Melo & HL Swinney, *Nature* **382**, 793, 1996.

19. PM Umbanhowar, F Melo & HL Swinney, *Physica A* **249**, 1, 1998.

20. T Shinbrot, *Nature* **389**, 574, 1997.

21. A Czirók & T Vicsek, *Physica A* **281**, 17, 2000.

22. ID Couzin & J Krause, *Adv. Stud. Behav.* **32**, 1, 2003.

23. K Libbrecht, *Am. Sci.* **95(1)**, 52, 2007.

24. Ignacio Rodriguez-Iturbe & Andrea Rinaldo, *Fractal River Basins. Chance and Self-Organization*, Cambridge University Press (Cambridge), 1997.

25. Michael Batty & Paul Longley, *Fractal Cities*, Academic Press (London), 1994.

26. EA Jagla & AG Rojo, *Phys. Rev. E* **65**, 026203, 2002.

27. Denis Weaire, *The Kelvin Problem*, Taylor & Francis (London), 1996.

28. Stephen Wolfram, *A New Kind of Science*, Wolfram Media Inc., 2002.

29. M Markus & B Hess, *Nature* **347**, 56, 1990.

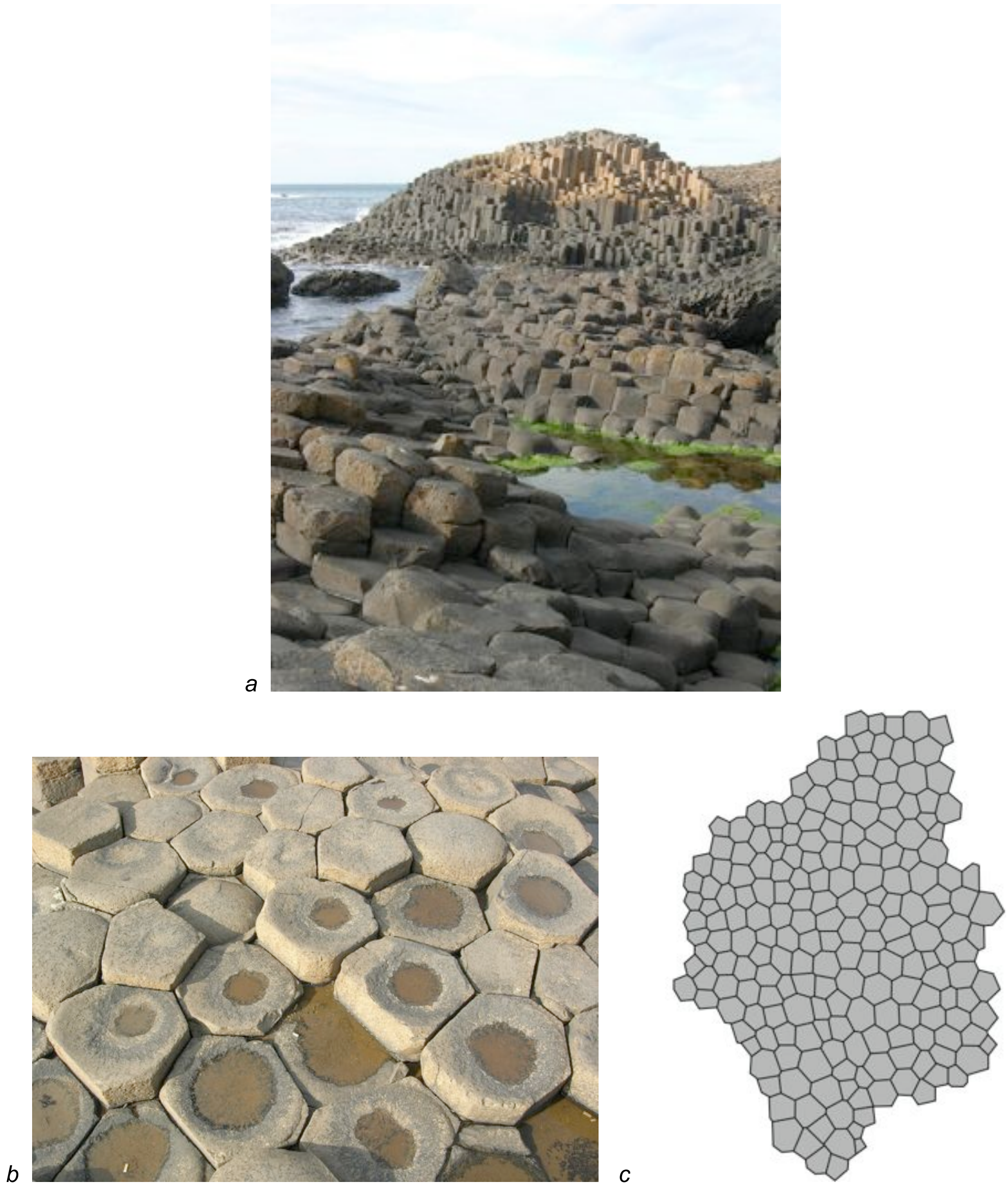


FIGURE 1   *a*, The Giant's Causeway in County Antrim, Ireland. *b*, A close-up of the prismatic cross-sections of rock columns. *c*, The crack network of a representative section, showing the quasi-hexagonal pattern. (Photos: *a*, *b,* Stephen Morris, University of Toronto.)

a b c

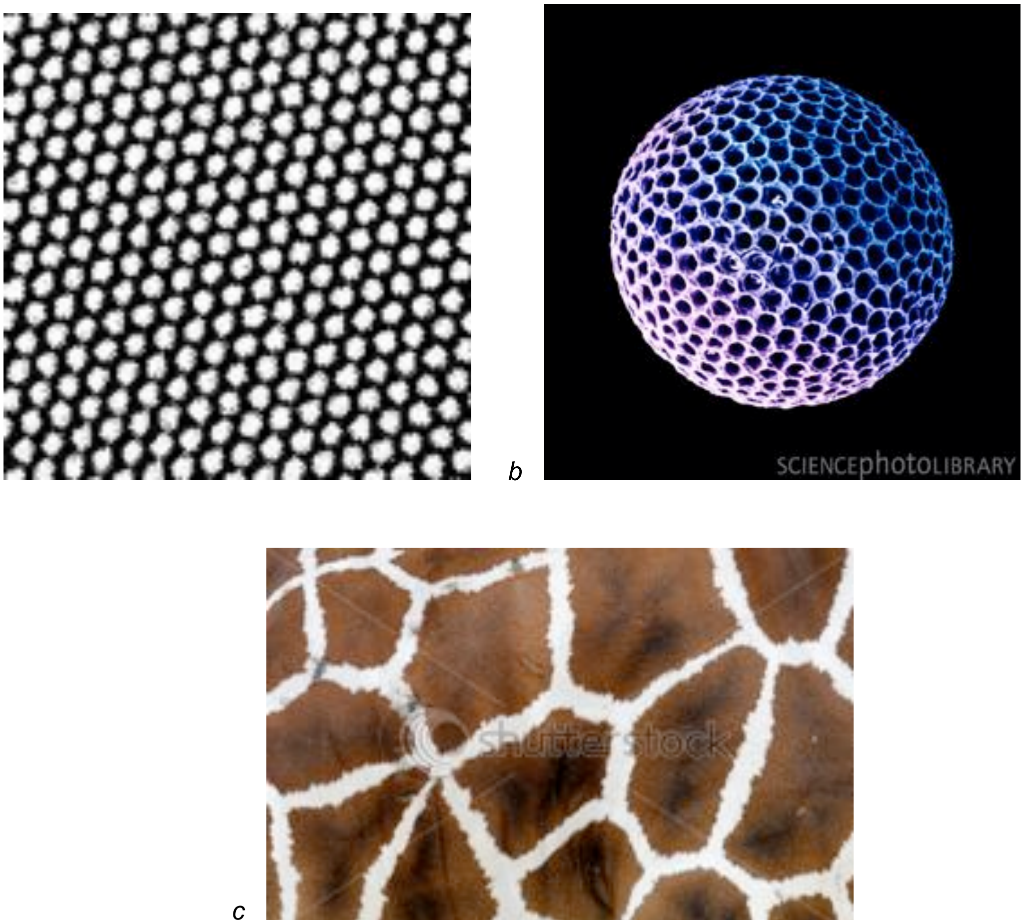


FIGURE 2   Spontaneously formed hexagonal and quasi-hexagonal patterns in convection (*a*), the shells of microscopic sea creatures (radiolarians) (*b*), and animal markings (*c*). (Photos: *a*, David Cannell, University of California at Santa Barbara; *b*, Dr Richard Kessel & Dr Gene Shih, Visuals Unlimited/Science Photo Library; *c*, Shutterstock/Nagel Photography.)

FIGURE 3   A snapshot of chemical-wave patterns in the oscillating Belousov-Zhabotinsky reaction. (Photo: Stephen Morris, University of Toronto.)

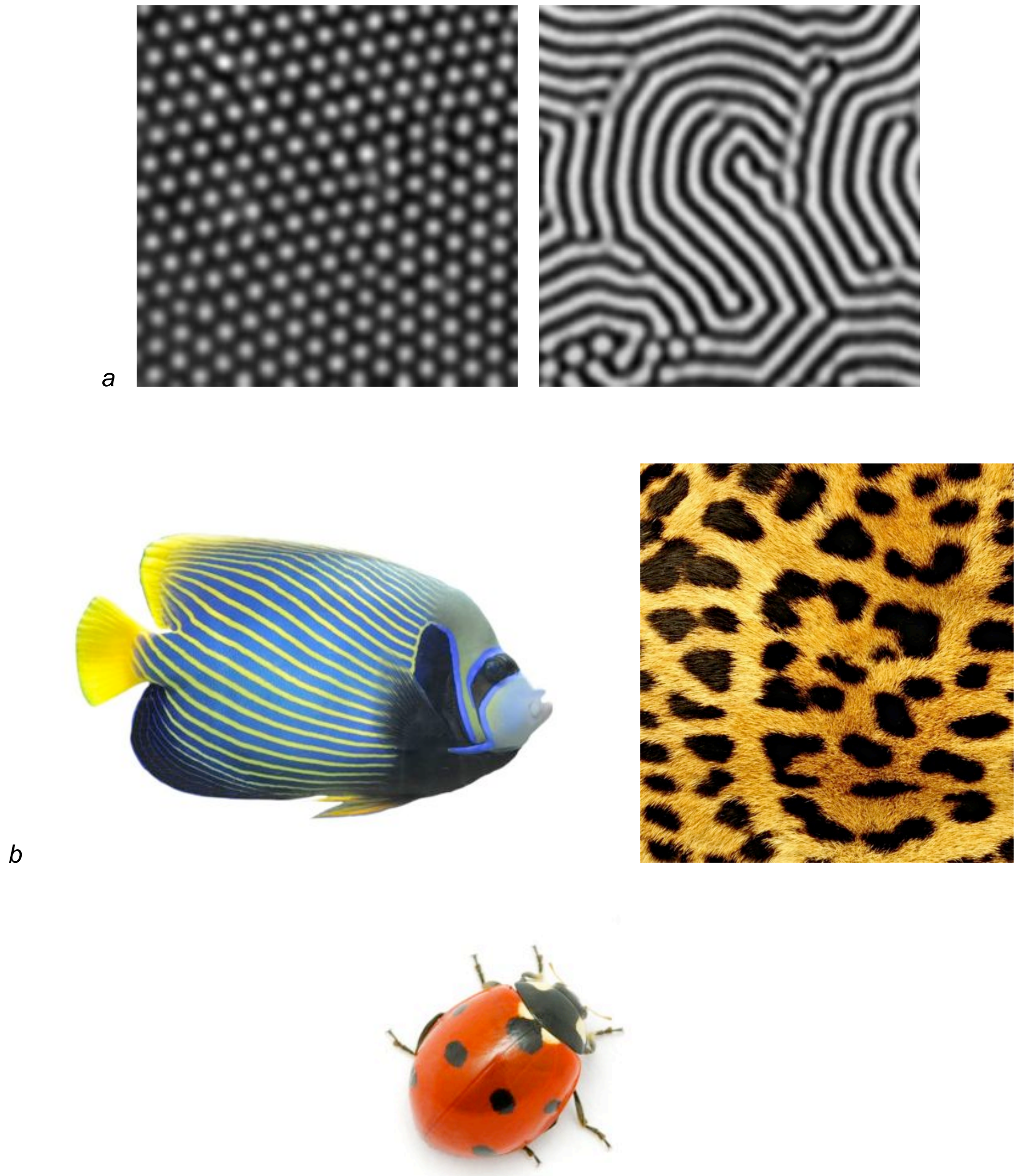
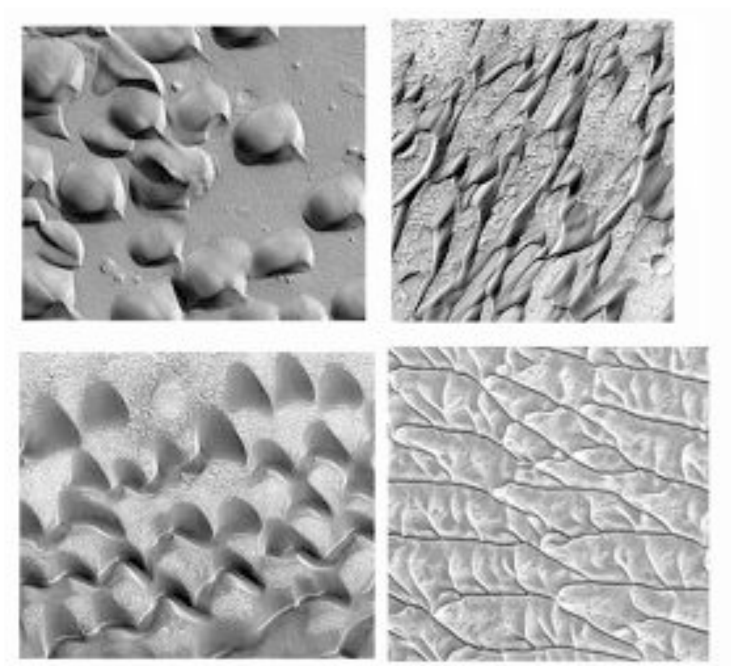


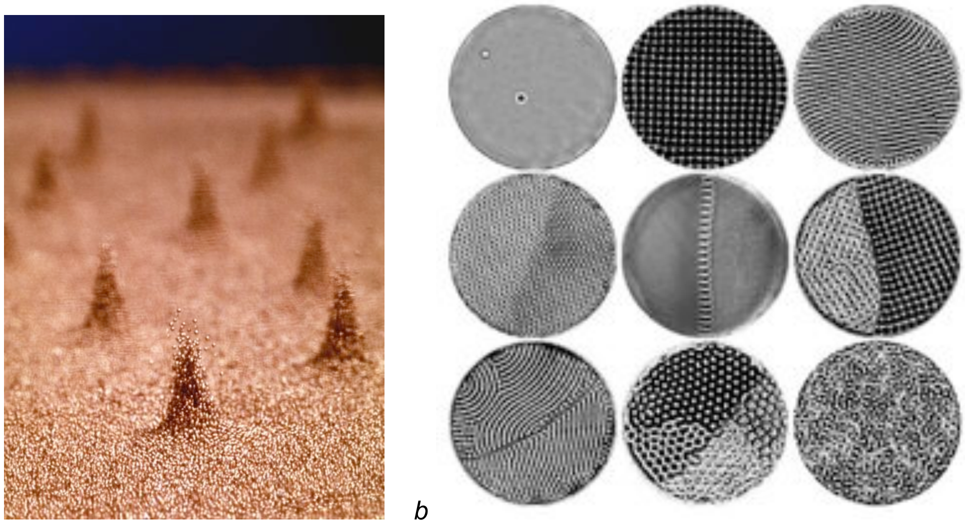
FIGURE 4   *a*, The generic forms of stationary chemical patterns (Turing patterns): spots and stripes. *b*, Schemes based on Turing patterns have been proposed to explain the markings on fish, wildcats and ladybirds. (Photos: *a*, Patrick De Kepper & Jacques Boissonade, University of Bordeaux; *b*, Shutterstock/cynoclub/tatniz/irin-k.)

a



b

FIGURE 5 Granular materials are prolific pattern-formers. Sand ripples seem to have the same sinuous form as a zebra’s stripes (*a*). Sand dunes are analogous structures on larger scales. On Mars, differences in wind speed, gravity and atmospheric pressure may create new dunes shapes (*b*). (Photos: *a*, Shutterstock/Darrenp; *b*, NASA.)



a *b*

FIGURE 6 Vertically vibrated grains will self-organize into isolated waves (oscillons; *a*) or a variety of patterns including extended standing waves (*b*). (Photos: Paul Umbanhowar, Northwestern University.)

Biography

Philip Ball is a freelance science writer, and previously an editor for *Nature*. He has published many books and articles on all areas of the natural sciences and their interactions with art and the wider culture, including the trilogy *Nature's Patterns: Shapes, Flow, Branches* (Oxford University Press, 2009).