Patterns in art and nature

Notes for a talk delivered at Dulwich Picture Gallery, May 2003

This talk is accompanied by a Powerpoint presentation

Fingal's Cave on the island of Staffa, near Mull in Scotland, has inspired artists (this is Turner) and composers – Felix Mendelsohn wrote his orchestral piece named after the cave in 1829. But it also made an impression on an awestruck Joseph Banks, president of the Royal Society, when he sailed to Staffa in 1772 during an expedition to Iceland. This is what he said:

> "Compared to this what are the cathedrals or palaces built by men! Mere models or playthings, as diminutive as his works will always be when compared with those of nature. What now is the boast of the architect! *Regularity*, the only part in which he fancied himself to exceed his mistress, Nature, is here found in her possession, and here it has been for ages undescribed."

Banks had noticed that the entrance to the cave was flanked by these great pillars of rock. Close up, you can see the regularity that Banks spoke about: hexagonal cross-sections.

Now, this of course has its counterpart on the west coast of Ireland itself: the Giant's Causeway in County Antrim, built in legend by the giant Finn MacCool.

There are something like 40,000 pillars of rock in the giant's Causeway, and they generally have this extraordinarily regular and geometric honeycomb structure. When we make an architectural pattern like this, it is through careful planning and construction, with each individual element cut to shape and laid in place. At Fingal's Cave and the Giant's Causeway, the forces of nature have conspired to produce such a pattern without, we must presume, any blueprint or foresight or design. This is an example of spontaneous pattern formation.

It seems almost incredible that a structure as regular as this can really arise unplanned in the natural world, and there are many examples of how scientists, natural historians and others have been totally misled by them by assuming automatically that they must be the product of human intelligence. The geometric columns of Fingal's cave and the Giant's Causeway are echoed elsewhere around the globe – for example, there is a formation called the Devil's Postpile in the Sierra Nevada in California. In 1998 there were reports of how a documentary film-maker in Nicaragua had discovered the ruins of an ancient lost city, consisting of 62 faceted columns of basaltic rock. The government had already dispatched tropps to guard the city against looters before geologists pointed out that this wasn't a human construction at all but something that occurs in nature.

But how are such things formed? In 1875 the Irish engineer and geologist Robert Mallett proposed that the pillars arise as molten volcanic rock cools. As it cools, it contracts – nearly every material does this, just as they expand when they get hot. But as it contracts everywhere at once, stresses build up everywhere in the rock, and it starts to crack. The same thing happens in mud as it dries, and the result is the familiar web of cracks that we see in parched river and lake beds [pic]. You can see that this is a fairly disorderly structure, and at first the solidifying volcanic rock looks like that too. But it cools and solidifies from the top downwards, so this pattern of cracks moves down through the rock as it cools layer by layer. Last year two scientists showed that in that situation the cracks redistribute the stresses so that they are more evenly distributed throughout the material, which means that the cracking pattern gets ever more regular as it moves downwards, eventually becoming organized into this kind of almost regular arrangement in which most of the polygons have five or six sides.

In other words, here's a situation in which forces push and pull until they create a structure that looks more or less the same everywhere: a pattern in which the basic elements repeat regularly. Something very similar can happen in a foam, which is really just a mass of bubbles packed together. A single layer of bubbles, all of the same size, gets organized by the surface tension of the liquid in the bubble walls so that it creates this hexagonal arrangement [pic]. By carefully controlling the size of the bubbles, we can make all kinds of other patterns from this fundamental tendency of bubbles to pack together hexagonally [pic – Weaire].

The hexagonal pattern of the Giant's Causeway and of bubbles natural pattern was already perfectly familiar to scientists in the eighteenth century. In fact, it had been known for several millennia: ever since humans began to keep bees. [pic]

This is the bee's honeycomb. The ancient Greeks presumed that bees must have some intrinsic sense of geometry to build a structure like this. The philosopher Pappus of Alexandria claimed that the bees had 'wisely selected' the cell shape that could hold most honey. Pliny tells us that some of the ancients devoted a lifetime's study to the mystery of how bees were able to build something so regular.

In the eighteenth century, a French scientist named R. A. F. de Réaumur suggested that the hexagonal pattern is the one that uses the least area of wall, and thus the least amount of wax, to divide up a given area into equal cells. But each cell has an end cap too, and the most economical shape for this is no easy thing to work out. De Réaumur asked the Swiss mathematician Samuel Koenig to solve that puzzle, and to do it Koenig needed to use the mathematical technique of calculus, which had only recently been devised by Isaac Newton and Gottfried Leibniz. He found that the best shape—this one here—was precisely the one the bees used. Now, it was too much for these men to suppose that the bees already knew the calculus before Newton, and so Bernard Fontenelle, secretary of the French Academy of Sciences, asserted that this must be the work of God: that the bees were "blindly using the highest mathematics by divine guidance and command".

So again, you see, this is where a consideration of natural pattern formation tended inevitably to lead. Pattern seemed to be the fingerprint of intelligent design. And to eighteenth-century scientists, if that intelligence was evidently not of human origin, it could come only from God.

Now, there are many, many different kinds of natural pattern. I'll have time to talk about only a few, but let me show you some of those about which I'll say nothing further: [animal pelts, sand dunes, phyllotaxis, spirals etc.]
What I want to touch on is not just how scientists have responded to the spontaneous pattern-forming propensity of nature, but how it has inspired artists from all eras.

The first person to truly address pattern formation as a general process that happens in nature was this chap, the Scottish zoologist D'Arcy Wentworth Thompson (1860-1948) [pic]. In 1917 Thompson published his masterpiece, *On Growth and Form*, which collected together all that was then known about pattern and form in nature in a stunning synthesis of biology, natural history, mathematics, physics and engineering. Stephen Jay Gould has called this "the greatest work of prose in twentieth-century science". Peter Medawar went even further, saying that it is "beyond comparison the finest work of literature in all the annals of science that have been recorded in the English tongue." In other words, *On Growth and Form* is beautifully written, and it is the bible of the scientific study of pattern formation. It is not an easy read, especially for the non-mathematical, and as the polymath Thompson was also a professor of classics, the book is apt to launch without translation or elaboration into ancient Greek, Latin, French, German and Italian. But it was a profound scientific achievement, and was decades ahead of its time.

D'Arcy Thompson had a mission, and that mission was to stem the tide of uncritical and indeed sometimes unscientific Darwinism that was washing throughout every corner of biological science. By the beginning of the twentieth century, the central idea of Darwin's theory was finally becoming appreciated, and it was an idea so powerful that some biologists had seemingly come to the conclusion that they needed nothing else. For every question that one could ask about biological shape and form, there seemed to be a single answer: natural selection. Why does this creature or that plant look the way it does? Natural selection! The form has obviously been selected because it is the one that is most evolutionarily fit, the one that does the job best. It had become tempting to see nature as an infinite palette of possibilities, from which only the best were selected.

D'Arcy Thompson pointed out that this couldn't be the whole story, because living organisms still have to function properly, which means that they must also satisfy the laws of mechanics. Just as you can't build a bridge or a skyscraper to an arbitrary design – it has to be mechanically stable in order to stay upright – so a plant or an elephant has to be designed so that it is robust and doesn't collapse. Thompson showed that biology is also about engineering.

He argued that many natural structures could be explained not by assuming that living organisms had constructed them painstakingly like bricklayers putting every element in place, but simply by appealing to physical forces. Thompson thought that the bee's honeycomb was really like a raft of bubbles, in that it was pulled into its regular hexagonal pattern by surface tension in the soft wax. Now, it turns out that he was wrong in this case – bees really are like bricklayers, carefully putting each flake of wax in its place and measuring the angles and so forth.

But D'Arcy Thompson was more or less right about another kind of hexagonal lattice structure: the skeletons of microscopic sea creatures called radiolarians [pic]. These little single-celled organisms have these extraordinary cage-like shells, called exoskeletons because they are essentially skeletons that appear *outside* the soft tissues. They are protective structures, like the hard shells of molluscs, often decorated with these prickly spines. These beautiful shapes are so small that they can only be seen under a microscope, and the first person to see and record them accurately was a nineteenth-century German biologist named Ernst Haeckel. D'Arcy Thompson proposed that these bony lattices were formed by the organisms blowing a foam of bubbles around themselves and then precipitating the hard mineral (in this case calcium carbonate, the same stuff that chalk is made from) in the junctions where one bubble met another. And that is indeed now how, basically, we think these exoskeletons are made.

Now, Haeckel wasn't just a biologist. He was also a wonderful draftsman, and early in his life even considered becoming a landscape artist. His work on the shapes and forms of the natural world, particularly of sea creatures, was hugely influential largely because he was able, in a time before the widespread use of photography, to illustrate so beautifully what he saw in the microscope [pic]. To Haeckel, structures like radiolarians were indeed a form of natural art, and between 1899 and 1904 he published a book of his drawings in ten instalments, called *Art Forms in Nature*, from which these images are taken. These works captured the imaginations not just of zoologists but of artists and designers, particularly those in the Art Nouveau movement and the German equivalent, the Jugendstil. The designer and architect René Binet corresponded with Haeckel and made direct use of his drawings, for example in his designs for furniture [pic, with Haeckel plate] and his entrance gate to the Paris World Exposition of 1900 [pic].

So, you see, it was no coincidence that the curved, biomorphic designs of Art Nouveau appeared in the late nineteenth and early twentieth centuries at just the same time as an appreciation of complex shape and form was starting to enter into science. Haeckel himself made no secret of the fact that his aesthetic response to these living structures provided a strong motivation for studying them. In a book cowritten with the zoologist W. Breitenbach in 1913, he said,

> Nature has scarcely produced anything so fragile and splendid in colour as these wonderful creatures… To laymen and inlanders, these daughters of the ocean seem mysterious and fantastic, and they do not really know what to make of them.

Haeckel was very much concerned with classifying organisms – he had a very important influence on Darwinian theory – and he could not but be influenced by the aesthetic superiority of some creatures over others, for example regarding complex, symmetrical jellyfish as 'higher' organisms than the mundane-looking sea cucumber. By the same token, it is striking how Art Nouveau artists saw in nature a kind of blueprint for creating forms and patterns that we would regard as beautiful and pleasing.

Now, here is another page of designs from René Binet, also inspired by the geometric patterns of radiolarians as portrayed by Haeckel – you might be able to see, for instance, the spikes protruding from the shells. But notice something else here. Binet has created these regular tiling patterns from hexagons. But he has also drawn two lone pentagons. Haeckel did indeed see pentagons in radiolarian shells [pic]; but Binet hasn't turned these into tiling patterns too. Why not?

The answer is that you can't tile with pentagons [pic – explain that there are always gaps]. In the hexagonal tilings of radiolarian exoskeletons, the pentagons play an essential role: they make the hexagonal sheets curved. The only way to fit a pentagon into a sheet of hexagons is to curve the sheet. This is what the exoskeletons need, so that they envelope the soft-tissued organism inside. Architects and designers have come to recognize this too. In the 1950s and 60s the American architect Richard Buckminster Fuller pioneered the use of so-called geodesic domes made from hexagonal elements [pic]; but his domes always contain a few strategically placed pentagons to induce curvature.

But as Binet no doubt realised, you can't make a perfect flat tiling pattern from pentagons or other regular five-sided shapes. Yet there is one artistic tradition that seems determined to defy this restriction: the geometrical designs of Islamic art. Here is an example of an Islamic design that contains many pentagonal elements. Admittedly, they are not simply pentagons packed together, because that is mathematically impossible. But Islamic artists, who were generally discouraged from making naturalistic representations of things by its prohibition in the Koran, were keen to examine how far they could push the restrictions set by geometry by incorporating into their regular patterns elements that had these mathematically 'forbidden'symmetries – not just pentagons but also eight-, ten- and twelve-sided objects, which also cannot be packed together in a perfect, gap-free tiling [pics].

Now, crystals are also a kind of geometric tiling. In crystalline materials, atoms are stacked regularly together in three dimensions, like cannonballs or oranges on a greengrocer's stall [pics]. This regularity is what creates the geometric facets of crystals [pic]. You can see that these arrangements too have particular kinds of symmetry: typically, packing a number of identical spheres together creates a hexagonal arrangement. Again, it is mathematically impossible to pack atoms together in a pentagonal arrangement, with so-called fivefold symmetry.

But in the 1980s, scientists discovered an apparently crystalline material that broke these rules. It was an alloy – a mixture – of the metals manganese and aluminium, and the techniques used for studying the atomic arrangement in solids seemed to show that this alloy had fivefold – in fact, tenfold – symmetry. Scientists were sure that this wasn't in fact possible, and so they refused to call this material a true crystal. They named it a quasicrystal.

And they soon noticed that when a quasicrystal is examined through a special kind of microscope that can see almost down to the level of individual atoms, the structure looks remarkably like an Islamic tiling pattern. Here the dark regions are where the atoms are, and you can't quite see them individually. But hopefully you can make out shapes with fivefold symmetry, such as pentagons of bright spots.

How can this substance be breaking the rules of geometry? Well, the answer is that it isn't. this might *look* like a regular, geometric pattern, but the fact is that it never repeats exactly. If I shift the pattern slightly, I can never get it to match up (demo with transparencies). Quasicrystals have an unusual arrangement of atoms that isn't quite regular, so that it can produce many individual, localized areas with fivefold symmetry without ever repeating itself exactly.

[pic] Here is another example of a hexagonal pattern in nature. Actually it is not really in nature, but in an artificial system consisting of a pan of liquid in which metal flakes are dispersed so that we can see the patterns of flow. The pan is being heated from below by standing it on a hotplate. When the plate gets hot enough, it sets up convection currents: the liquid at the bottom gets hot, expands slightly, and so becomes less dense, which makes it rise up to the top. Here the liquid loses its heat and cools down again, becoming more dense and sinking back to the bottom. So you get this circulation of liquid, called a convection current.

But the amazing thing is that these currents organize themselves spontaneously into this hexagonal pattern, with the fluid rising at the edges of the hexagons and sinking in the middle. We don't have to do anything special to the liquid to make this pattern: it simply appears once convection starts.

Hexagons aren't the only patterns that form in convection. Here are some more, revealed by a simple shadow technique in which liquid that is rising or sinking appears dark [pics]. We can get stripes, waves, spirals and disorderly patterns rather like fingerprints. Some structures like this do appear in the natural world. You might sometimes have noticed how clouds break up into small blobs or into parallel streaks called cloud streets. This is caused by convection currents in the air. And convection caused by the freezing and thawing of water in the ground in some Arctic regions has been suggested as the reason why stones get organized into regular shapes like this – this is Spitsbergen in Norway.

The flow of water has always presented a challenge to artists. How do you capture something so dynamic and ever-changing in a still picture? Rather different traditions have arisen in the West and the East. Western artists have tended to show flow as a play of light on the water surface. They paint choppy seas as a series of foam-flecked waves [pic]. But Chinese and Japanese artists don't strive for this kind of photographic snapshot realism. Instead, they attempt to portray the *essence* of flow, schematising it as a series of lines [pics]. The interesting thing about this way of representing flow is that it is very close to what scientists do. Those who study flow, called fluid dynamicists, generally show the structure of a flow using so-called streamlines, which, loosely speaking, show the trajectories that a floating object would follow in the flow [pic]. Water in Far Eastern art is typically depicted in terms of streamlines.

There *have* been attempts to illustrate flow this way in the West. Most famously, Leonardo da Vinci used this sort of approach in his studies of water flow, something that fascinated him [pic]. Again, we can see Leonardo striving to find the essential forms and patterns of flow beneath the ephemeral splashes and gleams. These flow forms are again reminiscent of the curved patterns of Art Nouveau, and were used a lot by the illustrator Arthur Rackham, whose works many of you no doubt saw here last year [pic].

Depicting flow using streamlines encourages artists to identify the regularities and the fundamental structures of flow. Scientists have discovered that these can be extraordinarily regular and beautiful. For example, what could be more archetypal of Art Nouveau than these structures? [pic] But they are simply the flow patterns that emerge as water flows past a cylinder, like a post standing upright in a stream. Here is an even more remarkable example of this pattern, which is called a Kármán vortex street after the Hungarian fluid dynamicist Theodore von Kármán [pic]. This is a satellite picture of clouds above an island of the Arctic ice cap. Convection has organized the clouds into these parallel stripes. But then the air flowing over the island has become disturbed at this point, presumably by a large pillar of ice, so that the wake has become this regular series of vortices.

If the flow past an obstacle like this gets too fast, the regularity of the vortex street is lost and the wake becomes turbulent (pic). That's what we see more often in nature, for example in the wake of the stanchions of a bridge across the Thames. Most rivers flow too fast to produce a vortex street. Describing turbulence scientifically is one of the most daunting challenges for scientists, and no one has yet managed to come up with a completely convincing theory for it.

But that needn't worry artists. The contemporary artist Susan Derges is fascinated by natural processes and how to capture them, and she has developed a method of photographing the complex structures of turbulent flow in streams and rivers [pic]. In these images she immersed huge sheets of photographic paper protected between glass plates just beneath the water surface of the River Taw in Devon at night, and illuminated them with a very brief flash of light. This reveals all the little peaks and troughs of surface waves as a kind of shadowgraph. It's interesting to note that Derges has herself studied Japanese art and so is familiar with its notion of distilling the universal from the particular.

The American artist Athena Tacha is also captivated by the patterns of fluid flow, as well as by the forms and regularities of nature more broadly – she says that D'Arcy Thompson's *On Growth and Form* has been an important source of inspiration. This is one of her works called *Dripping* (1969), which brings to mind D'Arcy Thompson's studies of droplets of ink descending through water and the shapes of sea creatures that these resemble [pic]. But many of Tacha's sculptures are more directly concerned with flow patterns, such as this design for a public sculpture called *Eddies/Interchanges (Homage to Leonardo)*, and this one called *Green Acres*, which was built in Trenton, NJ.

Some of the complex patterns of vortices seen in fluid flow might bring to mind images like this: [pic], which have become familiar from the work on fractals in the 1980s. Fractals are patterns that repeat again and again at ever decreasing scales, so that a small part of the structure reproduces the whole. This fractal pattern is generated using mathematics, but fractal-like structures do appear in nature – for example, in the way that trees fork repeatedly so that a twig resembles a branch which resembles the whole tree [pic]. Turbulent flow can have a kind of fractal character, in which the flow breaks up into vortices that are themselves decorated by smaller vortices and so on [pic]. This was acknowledged in a little rhyme coined by the scientist Lewis Fry Richardson in 1922, who modelled it on Jonathan Swift's poem about fleas:

> Big whirls have little whirls
> That feed on their velocity,
> And little whirls have lesser whirls
> And so on to viscosity

…which requires a fluid dynamicist's understanding of what is meant by viscosity in order to be fully understood.

Fractals were popularised and named by the mathematician Benoit Mandelbrot, but they were known in some sense to Richardson himself decades earlier. He pointed out that coastlines typically have the property that we now recognize as fractal, in that if you look at them ever more closely, you see much the same kind of irregularity [pic]. It is precisely this kind of complexity of form that has been explored by the Scottish contemporary artist Glen Onwin in his work *Nigredo* [pic]. This is a vast vat of black brine, installed in a derelict chapel in Halifax, on which some islands of wax are floating. The brine is so salty that some of the salt crystallizes out of the solution, and it does this around the edges of the wax islands, causing them to aggregate into irregular clusters that are forever changing as the salt precipitates and then re-dissolves. Onwin says that he is interested in exploring the interplay of processes at different scales, just as you find in fractal patterns:

> I want to work in a microscopic, macroscopic way, to get as much information from the larger image of the work as… from the detail. It is important that you know the detail is there, but you should stand back… and view the whole work with that knowledge.

I think it is fair to say that patterns and form have emerged as strong themes in contemporary art. For example, artists ranging from Bridget Riley to Damien Hirst to Andy Goldsworthy to the Japanese painter Yayoi Kusama have become interested in the effect of repeating motifs. Frankly, I find that the interest of these works is greatest when that repetition is not perfect and mechanical, but incorporates an element of randomness or surprise, just as it often does in nature. We can see that also in the folded or maze-like patterns seen in the brain, in so-called brain coral, and in some patterns in non-living systems such as the stripes and whorls seen in convection and in some magnetic materials [pics] – these motifs are picked up in works like Antony Gormley's *Bluestone* and Mona Hatoum's *Entrails Carpet*. Interestingly, they also put us in mind of some very ancient art like that found carved into the rocks of the Stone Age structure at Newgrange in Ireland, or that of aboriginal art [pics]. Which prompts me to finish with a question: are these natural patterns perhaps archetypal, and therefore destined to recur in the art of all ages?